Int. Conf. on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2009
B. Schrefler, E. Oñate and M. Papadrakakis (Eds)

THE COUPLING PROBLEM OF DIFFERENT THERMAL-HYDRAULIC MODELS ARISING IN TWO-PHASE FLOW CODES FOR NUCLEAR REACTORS

A. Ambroso*, C. Chalons†,‡, F. Coquel‡, E. Godlewski‡, F. Lagoutière†,‡, P.-A. Raviart‡ and N. Seguin‡

*DEN/DM2S/SFME, CEA-Saclay
91191 Gif-sur-Yvette cedex, France

†Université Paris Diderot-Paris 7
‡ Laboratoire Jacques-Louis Lions, Univ. Pierre et Marie Curie-Paris6, CNRS UMR 7598
Boîte courrier 187, 75252 Paris Cedex 05, France
e-mail: chalons@math.jussieu.fr, coquel@ann.jussieu.fr, godlewski@ann.jussieu.fr, lagoutie@math.jussieu.fr, pa@raviart.com, seguin@ann.jussieu.fr

Key words: Model Coupling, Two-Phase Flows, Boundary value problems, Numerics

Abstract. This paper briefly presents the research activity of our group on the coupling problem of different partial differential equations (PDE) at a fixed interface. Our motivation comes from the coupling of different two-phase flow codes that involve different PDE systems for simulating the components of a nuclear reactor.

1 INTRODUCTION

We have been considering in a series of papers the coupling of PDE systems, both from a theoretical and from a numerical point of view. For what concerns our industrial motivation, such a coupling arises for the simulation of nuclear reactors when different two-phase flow codes are used. In these codes, multiple modelling scales are applied to describe the flow. For instance, different thermal-hydraulic models can be used for each reactor component to take into account its specific behavior, or small scale models can be used, locally, to obtain a better resolution. When these models are put side to side, we face the problem of coupling. There is therefore a need to identify the nature of the information to be prescribed at a coupling interface, depending on the mathematical structure of the left and right PDE systems, to achieve a coherent description of the whole operating device.

This note is organized as follows. We first explain how the coupling problem is set from a mathematical point of view. We then briefly review some of the works that have been carried out by our group. See also http://www.ann.jussieu.fr/groupes/cea/ for details.

This work was partially supported by the NEPTUNE project[10], funded by CEA, EDF, IRSN and FRAMATONE-ANP.

2 THE COUPLING PROBLEM

In this section, we present the coupling problem focusing ourselves on systems of conservation laws having the same size and in one space dimension. The flow is assumed to be described by a first system (S_1) in $\mathcal{D}_1 = \{x < 0,\, t > 0\}$ and another system (S_2) in $\mathcal{D}_2 = \{x > 0,\, t > 0\}$. We look for a solution $\mathbf{u} : (x,t) \in I\!R \times I\!R_+ \to \mathbf{u}(x,t) \in \Omega \in I\!R^n$, $n \geq 1$ of the following coupling problem

$$(S_1) \quad : \quad \partial_t \mathbf{u} + \partial_x \mathbf{f}_1(\mathbf{u}) = 0, \quad (x,t) \in \mathcal{D}_1, \tag{1}$$

$$(S_2) \quad : \quad \partial_t \mathbf{u} + \partial_x \mathbf{f}_2(\mathbf{u}) = 0, \quad (x,t) \in \mathcal{D}_2, \tag{2}$$

with initial condition $\mathbf{u}(x,0) = \mathbf{u}_0(x), \quad x \in I\!R$. We assume in addition that the flux functions $\mathbf{f}_1 : \Omega \to I\!R$ and $\mathbf{f}_2 : \Omega \to I\!R$ are smooth and that systems (S_1) and (S_2) are hyperbolic on the set of states Ω. At interface $x = 0$, we add some coupling model.

Interface models can be gathered into two groups. The first one, the so-called *state coupling*, aims at imposing the continuity of a set of variables $\mathbf{v} = \varphi(\mathbf{u})$. It writes

$$\mathbf{v}(0^-,t) = \mathbf{v}(0^+,t), \tag{3}$$

with $\mathbf{v}(0^-,t) = \varphi_1(\mathbf{u}(0^-,t))$ and $\mathbf{v}(0^+,t) = \varphi_2(\mathbf{u}(0^+,t))$. Here, $\mathbf{u}(0^-,t)$ (respectively $\mathbf{u}(0^+,t)$) denotes the left (resp. right) trace of the solution, and φ_1 and φ_2 define two admissible changes of variables $\mathbf{u} \to \mathbf{v}$ associated with (S_1) and (S_2) respectively.
The second one, the so-called *flux coupling*, aims at imposing the continuity of the flux across the interface, which writes

$$\mathbf{f}_1(\mathbf{u}(0^-,t)) = \mathbf{f}_2(\mathbf{u}(0^+,t)). \tag{4}$$

Let us first notice that the flux functions $\mathbf{f}_1$ and $\mathbf{f}_2$ are generally not invertible, so that the flux coupling actually differs from the state coupling. Note also that for the flux coupling, $\mathbf{u}$ is a conservative variable in the whole domain $I\!R \times I\!R^+$, not only in both regions $\mathcal{D}_1$ and $\mathcal{D}_2$ like in the state coupling.

The state coupling. Due to the hyperbolic nature of both systems (S_1) and (S_2), the continuity property (3) is generally too restrictive and must be understood in a weak sense. More precisely, we follow the weak boundary conditions principle proposed by Dubois and LeFloch[7] and Godlewski and Raviart[8] (see also[9,2]). For $i = 1, 2$, we denote $\mathbf{u}_i(x/t; \mathbf{u}_l, \mathbf{u}_r)$ the solution to the Riemann problem

$$\left\{ \begin{array}{l} \partial_t \mathbf{u} + \partial_x \mathbf{f}_i(\mathbf{u}) = 0, \quad x \in I\!R,\ t > 0, \\ \mathbf{u}(x,0) = \left\{ \begin{array}{ll} \mathbf{u}_l & \text{if} \quad x < 0, \\ \mathbf{u}_r & \text{if} \quad x > 0, \end{array} \right. \end{array} \right. \tag{5}$$

and by $\mathbf{v}_i(x/t;\mathbf{v}_l,\mathbf{v}_r)$ the solution of this problem with respect to the $\mathbf{v}$ variable, *i.e.*

$$\mathbf{v}_i(x/t;\mathbf{v}_l,\mathbf{v}_r)=\varphi_i(\mathbf{u}_i(x/t;\varphi_i^{-1}(\mathbf{v}_l),\varphi_i^{-1}(\mathbf{v}_r))). \tag{6}$$

We then define the sets of admissible traces in 0^- and 0^+ for systems (S_1) and (S_2) :

$$\mathcal{O}_1(\mathbf{b})=\left\{\mathbf{v}_1(0^-;\mathbf{v},\mathbf{b});\mathbf{v}\in\varphi_1(\Omega)\right\},\quad \mathcal{O}_2(\mathbf{b})=\left\{\mathbf{v}_2(0^+;\mathbf{b},\mathbf{v});\mathbf{v}\in\varphi_2(\Omega)\right\},$$

where $\mathbf{b}\in\Omega$ is a given boundary condition. The weak formulation of (3) then writes

$$\mathbf{v}(0^-,t)\in\mathcal{O}_1(\mathbf{v}(0^+,t)),\quad \mathbf{v}(0^+,t)\in\mathcal{O}_2(\mathbf{v}(0^-,t)).$$

This strategy allows to ensure *as far as possible* the continuity of the vector $\mathbf{v}$ but this is not always the case. Indeed when the coupling interface becomes characteristic (*i.e.* when the characteristic speeds of (S_1) and (S_2) equal zero), some components of the vector $\mathbf{v}$ may be discontinuous across this interface.

The flux coupling. Imposing the continuity of the flux function across the coupling interface is often natural from a physical point of view. In this case, the corresponding coupling problem takes the form of a global problem that writes as follows :

$$\partial_t\mathbf{u}+\partial_x\mathbf{f}(\mathbf{u},x)=0,\quad x\in I\!R,\ t>0, \tag{7}$$

with $\mathbf{f}(\mathbf{u},x)=(1-H(x))\mathbf{f}_1(\mathbf{u})+H(x)\mathbf{f}_2(\mathbf{u})$. Here H denotes the Heaviside function. Note that generally speaking, the flux continuity (4) does not imply the continuity of $\mathbf{u}$.

It is important to notice at this stage that from a theoretical point of view, the choice between these two families of interface models must be done according to the physics of the underlying simulation. From a numerical point of view, the point is to design numerical strategies that account for the chosen interface model.

3 BRIEF REVIEW OF OUR CONTRIBUTIONS

The proposed definition of the interface models leads directly to the mathematical question of existence and uniqueness of a solution to the coupling problem.

In[6], we investigate this issue for the one dimensional state coupling of two gas dynamics systems in Eulerian coordinates. Several examples of solutions, either continuous or discontinuous at the coupling interface, are constructed.

In[2], the special structure of Lagrangian systems enabled us to deal with the coupling conditions at interface, when a special set of transmitted variables is chosen, for rather general fluid systems.

In another direction, we have used relaxation systems to achieve a conservative numerical coupling of two Euler systems that avoids resonance[5].

In[1], we consider the *numerical* coupling of two homogeneous models used for the description of non isothermal compressible two-phase flows. More precisely, we concentrate on the numerical coupling of the homogeneous equilibrium model and the homogeneous relaxation model. Several finite volume methods are presented to achieve, at the numerical level, the desired coupling.

More recently, we have started to study the coupling of a two fluid-two pressure diphasic model and its drift-flux asymptotic limit which is of primary interest for the simulation of a global nuclear reactor. See for instance[3,4] and the references therein.

REFERENCES

[1] A. Ambroso, C. Chalons, F. Coquel, E. Godlewski, F. Lagoutière, P.-A. Raviart, N. Seguin. *The coupling of homogeneous models for two-phase flows.* Int. Journal for Finite Volume, **4**(1) (2007).

[2] A. Ambroso, C. Chalons, F. Coquel, E. Godlewski, F. Lagoutière, P.-A. Raviart, N. Seguin. *Coupling of general Lagrangian systems*, Mathematics of Computation, 77, pp 909-941 (2008).

[3] A. Ambroso, C. Chalons, F. Coquel, T. Galié, E. Godlewski, P.-A. Raviart, N. Seguin. *The drift-flux asymptotic limit of barotropic two-phase two-pressure models.* Comm. Math. Sci., 6, pp 521-529 (2008).

[4] A. Ambroso, C. Chalons, F. Coquel, T. Galié. *Relaxation and numerical approximation of a two fluid two pressure diphasic model.* to appear in ESAIM:M2AN

[5] A. Ambroso, C. Chalons, F. Coquel, E. Godlewski, F. Lagoutière, P.-A. Raviart, N. Seguin. *Relaxation methods and coupling procedure.* International Journal for Numerical Methods in Fluids, 56, issue 8, pp 1123-1129 (2008).

[6] C. Chalons, P.-A. Raviart, N. Seguin. *The interface coupling of the gas dynamics equations.* Quarterly of Applied Mathematics, 66, no. 4, pp 659-705 (2008).

[7] F. Dubois, P.-G. LeFloch. *Boundary conditions for nonlinear hyperbolic systems of conservation laws*, J. Differential Equations, 71, 93–122 (1988).

[8] E. Godlewski, P.-A. Raviart. *The numerical interface coupling of nonlinear hyperbolic systems of conservation laws. I. The scalar case.*, Numer. Math., 97(1), pp 81–130 (2004).

[9] E. Godlewski, K.-C. Le Thanh, P.-A. Raviart. *The numerical interface coupling of nonlinear hyperbolic systems of conservation laws: II. The case of systems.* ESAIM:M2AN, 39, 49–692 (2005).

[10] A. Guelfi, D. Bestion, M. Boucker, P. Boudier, P. Fillion, M. Grandotto, J.-M. Hérard, E. Hervieu, P. Péturaud. *NEPTUNE: A New Software Platform for Advanced Nuclear Thermal Hydraulic.* Nuclear Science and Engineering, 156, pp 281-324 (2007).