Extension of interface coupling to general Lagrangian systems

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Summary. We study the coupling of two gas dynamics systems in Lagrangian coordinates at the interface $x = 0$. The coupling condition was formalized in [9], [10] by requiring that two boundary value problems should be well-posed, and it yields as far as possible the continuity of the solution at the interface. In this work we prove that we may choose the variables we transmit and extend the theory to Lagrangian systems of different sizes. The coupling condition is expressed in terms of Riemann problems, which is well suited for the numerical methods we are interested in implementing. Moreover this formalism is well adapted to Lagrangian systems since the sign of the wave speeds is known, which enables us to solve the coupled Riemann problem.

1 Introduction

We are interested in the study of the coupling of two different hyperbolic systems at a fixed interface. In [9], a new coupling condition (CC in the sequel) is defined which results by expressing that two boundary value problems should be well-posed and the approach is justified in the scalar case. This CC resumes to impose as far as possible the continuity of the solution at the interface. The case of linear systems and ideas for the Euler system follow in [10]. Here, we show that in fact we can choose the set of dependent variables which is transmitted and illustrate the result with systems in Lagrangian coordinates for which the solution of the coupled Riemann problem is given explicitly and illustrated numerically. We have chosen to express the boundary conditions in terms of Riemann problems (see [7]). This approach is well suited for the numerical methods we are interested in implementing and linked to the theoretical results concerning the convergence of the two-flux method in the scalar case [9].

We first describe the theoretical settings and precise our notations. The case of of the $p-$system is then detailed, with numerical illustrations and Lagrangian systems are considered in the following sections.

1.1 Coupling procedure

Let $\Omega \subset \mathrm{R}^p$ be the set of states and let $\mathbf{f}_\alpha, \alpha = L, R$, be two 'smooth' functions from Ω into R^p. Given a function $\mathbf{u}_0 : x \in \mathrm{R} \to \mathbf{u}_0(x)$, we want to find a function $\mathbf{u} : (x,t) \in \mathrm{R} \times \mathrm{R}_+ \to \mathbf{u}(x,t) \in \Omega$ solution of

$$\partial_t \mathbf{u} + \partial_x \mathbf{f}_L(\mathbf{u}) = \mathbf{0}, \quad x < 0,\ t > 0, \tag{1}$$

$$\partial_t \mathbf{u} + \partial_x \mathbf{f}_R(\mathbf{u}) = \mathbf{0}, \quad x > 0,\ t > 0, \tag{2}$$

satisfying the initial condition $\mathbf{u}(x,0) = \mathbf{u}_0(x), x \in \mathrm{R}$, and at the interface $x = 0$, a coupling condition CC which we now describe. We have chosen this CC in order to obtain two well-posed initial boundary-value problems in $x > 0$, $t \geq 0$ and in $x < 0$, $t \geq 0$. This means that the trace $\mathbf{u}(0-,t)$ (resp. $\mathbf{u}(0+,t)$), $t \geq 0$, should be an admissible boundary condition at $x = 0$ for the system in $x > 0$ (resp. $x < 0$). We will assume that the systems are hyperbolic, i.e. for $\alpha = L, R$, the Jacobian matrix $\mathbf{f}'_\alpha(\mathbf{u})$ of $\mathbf{f}_\alpha(\mathbf{u})$ is diagonizable with real eigenvalues $\lambda_{\alpha,k}(\mathbf{u})$ and corresponding eigenvectors $\mathbf{r}_{\alpha,k}(\mathbf{u}), 1 \leq k \leq p$. Then we introduce the solution of the Riemann problem for the system associated to the flux $\mathbf{f}_\alpha$,

$$\mathbf{u}(x,t) = \mathbf{W}_\alpha(x/t; \mathbf{u}_L, \mathbf{u}_R)$$

i.e. the solution of $\partial_t \mathbf{u} + \partial_x \mathbf{f}_\alpha(\mathbf{u}) = \mathbf{0}$ with initial condition

$$\mathbf{u}(x,0) = \begin{cases} \mathbf{u}_L, & x < 0, \\ \mathbf{u}_R, & x > 0. \end{cases} \tag{3}$$

We set for all $\mathbf{b} \in \Omega$,

$$\begin{aligned} \mathcal{O}_L(\mathbf{b}) &= \{\mathbf{w} = \mathbf{W}_L(0-; \mathbf{u}_\ell, \mathbf{b}); \mathbf{u}_\ell \in \Omega\} \\ \mathcal{O}_R(\mathbf{b}) &= \{\mathbf{w} = \mathbf{W}_R(0+; \mathbf{b}, \mathbf{u}_r); \mathbf{u}_r \in \Omega\} \end{aligned} \tag{4}$$

and we define admissible boundary conditions of the form $\mathbf{u}(0-,t) \in \mathcal{O}_L(\mathbf{b}(t))$, $t > 0$, for (1) (resp. $\mathbf{u}(0+,t) \in \mathcal{O}_R(\mathbf{b}(t))$, $t > 0$, for (2)). Hence natural coupling conditions for problem (1)–(2) consist in requiring

$$\begin{cases} \mathbf{u}(0-,t) \in \mathcal{O}_L(\mathbf{u}(0+,t)), \\ \mathbf{u}(0+,t) \in \mathcal{O}_R(\mathbf{u}(0-,t)). \end{cases} \tag{5}$$

The approach is thoroughly justified in the scalar case [9] and for linear systems [10]. In [9] it is shown that this is indeed a 'reasonable' way of coupling two conservation laws in the sense that, in meaningful situations, the coupled problem has a unique solution and the 'natural' numerical upwind scheme (the so called two-fluxes scheme) converges to this solution. Condition (5) resumes in a number of cases to the continuity of the solution at the interface

$$\mathbf{u}(0-,t) = \mathbf{u}(0+,t). \tag{6}$$

Thus we may interpret the coupling condition as a way of ensuring in a weak sense the continuity, we will say the *transmission* of the conservative variables.

1.2 Numerical coupling

We use a finite volume method for each system (1), (2). Let Δx, Δt, denote the uniform space and time steps, $\mu = \Delta t/\Delta x$, $t_n = n\,\Delta t$, $n \in \mathrm{N}$, $C_{j+1/2} = (x_j, x_{j+1})$, the cell with center $x_{j+1/2} = (j+1/2)\,\Delta x$, $j \in \mathrm{Z}$. The initial condition is discretized by $\mathbf{u}^0_{j+1/2} = \frac{1}{\Delta x}\int_{C_{j+1/2}} \mathbf{u}_0(x)dx$, $j \in \mathrm{Z}$. For the numerical coupling, we are given two numerical fluxes $\mathbf{g}_L$, $\mathbf{g}_R$ ($\mathbf{g}_\alpha$ is consistent with $\mathbf{f}_\alpha$) corresponding to 3-point monotone schemes (under some CFL condition), we set $\mathbf{g}^n_{\alpha,j} = \mathbf{g}_\alpha(\mathbf{u}^n_{j-1/2}, \mathbf{u}^n_{j+1/2})$ and define the scheme by

$$\mathbf{u}^{n+1}_{j-1/2} = \mathbf{u}^n_{j-1/2} - \mu\left(\mathbf{g}^n_{L,j} - \mathbf{g}^n_{L,j-1}\right), \quad j \le 0, n \ge 0,$$

$$\mathbf{u}^{n+1}_{j+1/2} = \mathbf{u}^n_{j+1/2} - \mu\left(\mathbf{g}^n_{R,j+1} - \mathbf{g}^n_{R,j}\right), \quad j \ge 0, n \ge 0,$$

(see also [1] in another context). So we have one fixed interface at $x = 0$ and two fluxes $\mathbf{g}^n_{\alpha,0}$. The choice $\mathbf{g}^n_{\alpha,0} = \mathbf{g}_\alpha(\mathbf{u}^n_{-1/2}, \mathbf{u}^n_{1/2}), \alpha = L, R$ corresponds to transmit the conservative variables. Namely, if $j \ge 0$, the scheme with flux $\mathbf{g}_R$ approximates the IBVP (1) with initial condition $u(x,0) = u_0(x), x > 0$ and for boundary condition at $x = 0$, the scheme takes $\mathbf{u}^n_{-1/2}$. Since $\mathbf{g}^n_{L,0} \neq \mathbf{g}^n_{R,0}$, it is a nonconservative numerical approach, as for the continuous problem. For example, the flux at the boundary with Godunov's scheme is $\mathbf{g}^n_{R,0} = \mathbf{f}_R(\mathbf{W}_R(0+; \mathbf{u}^n_{-1/2}, \mathbf{u}^n_{1/2}))$.

1.3 Choice of transmitted variables

When dealing with physical systems, we may prefer to transmit not the conservative variables but the *physical* variables, or even the flux. Now, assume that there exists a change of variables $\mathbf{v} \to \mathbf{u} = \varphi_\alpha(\mathbf{v}); \alpha = L, R$ from some set $\Omega_{\mathbf{v}} \subset \mathrm{R}^p$ onto Ω such that $\varphi'_\alpha(\mathbf{v})$ is an isomorphism of R^p. Then if $\mathbf{c}$ is a given boundary *physical* data, setting $\mathbf{b}_\alpha = \varphi_\alpha(\mathbf{c})$, we define $\mathcal{O}_L(\mathbf{b}_L)$ and $\mathcal{O}_R(\mathbf{b}_R)$ which are admissible boundary sets for the systems (1) and (2) respectively. Thus we now require

$$\begin{cases} \mathbf{u}(0-,t) \in \mathcal{O}_L(\varphi_L(\mathbf{v}(0+,t)), \\ \mathbf{u}(0+,t) \in \mathcal{O}_R(\varphi_R(\mathbf{v}(0-,t)). \end{cases} \tag{7}$$

Since $\varphi_L(\mathbf{v}(0+,t)) \neq \varphi_R(\mathbf{v}(0+,t)) = \mathbf{u}(0+,t)$, the boundary sets in (5) and (7) are a priori distinct. Conditions (7) will ensure whenever possible the transmission of *physical* variables and their continuity instead of (6)

$$\mathbf{v}(0-,t) = \mathbf{v}(0+,t). \tag{8}$$

We are going to illustrate the two choices in the coupling procedure on the $p-$system and then for the full Euler system in Lagrangian coordinates. On the one hand, it is a simplified model of what we get when coupling more complex models associated to distinct systems whose closure laws are not always

compatible, as will happen for instance in the context of thermal-hydraulics. On the other hand, the analysis will justify the use of Lagrange+projection schemes when coupling systems in Eulerian coordinates at a fixed interface. Note however that for the Euler system in Lagrangian coordinates, the interface is characteristic and corresponds to a contact discontinuity. Hence, the coupling does not yield the continuity (6) or (8) for all the components. In our case, physical arguments, such as the continuity of some primitive quantities (for instance velocity and pressure) help defining the transmission. However, both theoretical considerations and numerical results obtained on some significant tests when coupling Euler systems (see [3][4]) will prove that if several CC based on continuity arguments are feasible, one cannot maintain all the conservation properties and we must choose which we want to be preserved.

2 Coupling two $p-$systems

We consider two systems (1) and (2) with

$$\begin{cases} \mathbf{u} = (\tau, v)^T, \tau > 0 \\ \mathbf{f}_L(\mathbf{u}) = (-v, p)^T,\ p = p_L(\tau), \\ \mathbf{f}_R(\mathbf{u}) = (-v, p)^T,\ p = p_R(\tau). \end{cases} \tag{9}$$

The two systems differ by the pressure law $p = p(\tau)$. We assume that $p'_\alpha < 0, p''_\alpha > 0$. The eigenvalues are $\pm\sqrt{-p'_\alpha}$.

We first transmit the conservative variables (τ, v). The study of the Riemann problem is needed in order to express the CC. We denote by $\mathcal{C}^i_\alpha(\mathbf{u}_-)$ the $i-$wave curve, i.e., the set of states that can be connected to a given state $\mathbf{u}_-$ by a $i-$ wave (either rarefaction or admissible shock) relative to the $p-$system with flux $\mathbf{f}_\alpha$. Expressing (5) gives that $\mathbf{u}(0-)$ is connected to $\mathbf{u}(0+)$ by a $2-L$ (positive) wave which means $\mathbf{u}(0+) \in \mathcal{C}^2_L(\mathbf{u}(0-))$ and similarly (for the right condition) by a $1-R$ (negative) wave. Thus $\mathbf{u}(0+) \in \mathcal{C}^2_L(\mathbf{u}(0-)) \cap \mathcal{C}^1_R(\mathbf{u}(0-))$ and $\mathbf{u}(0+) = \mathbf{u}(0-)$ because it is well known that the two wave curves intersect at only one point in the plane (τ, v) (see for instance [8]).

Now the IBVP's in both half planes are also well posed if one 'imposes' a given (v, p) on $x = 0$. Indeed, by assumption $p'_\alpha < 0$, we can define its inverse mapping $\tau_\alpha(p)$ for $\alpha = L, R$. Setting $\mathbf{v} = (v, p)^T$, we have an admissible change of variables: $\mathbf{u} = \varphi_\alpha(\mathbf{v})$ where

$$(v, p) \to \varphi_\alpha(v, p) \equiv (\tau, v) \tag{10}$$

is simply defined by $\tau = \tau_\alpha(p)$, for instance if $p_\alpha(\tau) = \tau^{-\gamma_\alpha}$, $\tau_\alpha(p) = p^{-1/\gamma_\alpha}$.

We now transmit this set of variables (v, p). Expressing the coupling condition (7) yields that $\varphi_R(\mathbf{v}(0-, t))$ is connected to $\mathbf{u}(0+, t) = \varphi_R(\mathbf{v}(0+, t))$ by a $1 - R$ wave. We can parametrize the wave curves by p and project them onto the $(v, p)-$plane (see [8], Chapter I, section 7). If the $1 - R$ wave curve is $\mathcal{C}^1_R(\mathbf{u}(0-)) = \{(\tau, v); v = \Psi_{1,R}(\tau)\}$, then $\tilde{\mathcal{C}}^1_R(\mathbf{v}(0-)) = \{(v, p); v =$

$\Psi_{1,R}(\tau_R(p))\} = \{(v,p); \varphi_R(v,p) \in \mathcal{C}^1_R(\mathbf{u}(0-))\} = \varphi_R^{-1}(\mathcal{C}^1_R(\mathbf{u}(0-)))$ is its representation in the $(v,p)-$coordinates, we then have $\mathbf{v}(0+) \in \tilde{\mathcal{C}}^1_R(\mathbf{v}(0-))$. Similarly, $\mathbf{u}(0-,t) \in \mathcal{O}_L(\varphi_L(\mathbf{v}(0+,t)))$ yields $\mathbf{v}(0+) \in \tilde{\mathcal{C}}^2_L(\mathbf{v}(0-))$. We get $\mathbf{v}(0+) \in \tilde{\mathcal{C}}^1_R(\mathbf{v}(0-)) \cap \tilde{\mathcal{C}}^2_L(\mathbf{v}(0-))$ and it is easily proved that the two curves intersect at only one point in the plane (v,p) so that $\mathbf{v}(0+) = \mathbf{v}(0-)$. Hence we do have continuity of v, p, but not of τ since $\tau(0+) = p(0+)^{-1/\gamma_R} \neq p(0-)^{-1/\gamma_L} = \tau(0-)$.

Let us illustrate the results on the solution of a Riemann problem (the exact solution is known). We take a uniform grid, with 150 meshes and a first-order explicit Roe-type coupled scheme, the CFL is 0.5. The two pressure laws are $p_\alpha(\tau) = \tau^{-\gamma_\alpha}$ with $\gamma_L = 1.4, \gamma_R = 1.6$, and we represent in this order τ, v and p at a given time t (exact and approximate solution). We note in

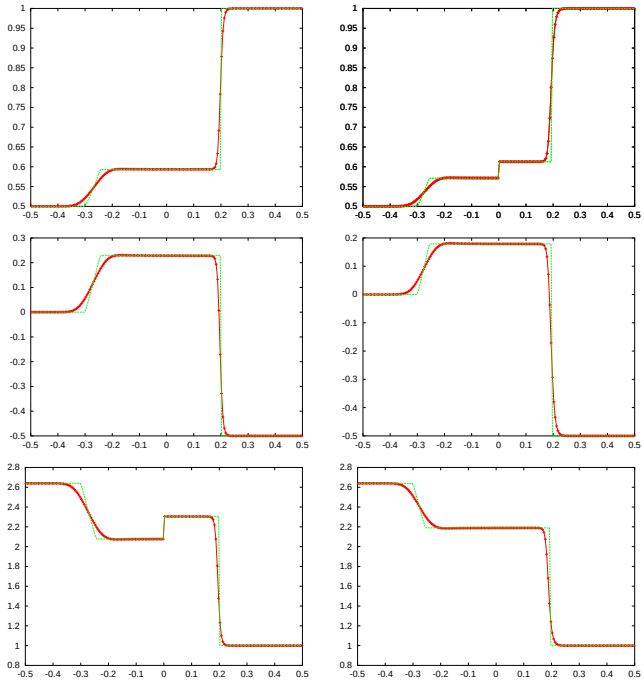

Fig. 1. Transmission of $\mathbf{u} = (\tau, v)$ left vs $\mathbf{v} = (v,p)$ right for the coupled $p-$system

fig.1, left part, the continuity of τ, v the discontinuity of p at $x = 0$ while in the right part we note the discontinuity of τ and the continuity of v, p.

3 Coupling two Euler systems in Lagrangian coordinates

We consider the system of gas dynamics in Lagrangian coordinates

$$\partial_t \mathbf{u} + \partial_x \mathbf{f}(\mathbf{u}) = \mathbf{0}, \mathbf{u} = (\tau, v, e)^T,\ \mathbf{f}(\mathbf{u}) = (-v, p, pv)^T. \tag{11}$$

In (11), x stands for a mass variable, τ denotes the specific volume, v the velocity, $e = \varepsilon + \frac{1}{2}v^2$ the specific total energy, ε the specific internal energy, and we assume that the pressure p is a given function $p = p(\tau, \varepsilon)$. We study the coupling of two such systems at $x = 0$ thus at a contact discontinuity separating two fluids with different equations of state $p = p_\alpha(\tau, \varepsilon),\ \alpha = L, R$. We denote by

$$\mathbf{f}_\alpha(\mathbf{u}) = (-v, p, pv)^T,\ p = p_\alpha(\tau, \varepsilon), \alpha = L, R, \tag{12}$$

the corresponding flux functions. The eigenvalues of the Jacobian matrix of $\mathbf{f}(\mathbf{u})$ are $\lambda_1(\mathbf{u}) = -C < \lambda_2 = 0 < \lambda_3(\mathbf{u}) = C$, where $C = \sqrt{-p_\tau + pp_\varepsilon}$ denotes the Lagrangian sound speed. In this case, the interface $x = 0$ is characteristic ($\lambda = 0$ is an eigenvalue) hence, in general, the coupling does not yield the continuity of (6) nor of (8). However we have for each system one strictly positive and one strictly negative eigenvalue and we will see that it yields the continuity of a subset of two variables. When coupling the two systems (1) and (2) with $\mathbf{f}_\alpha$ given by (12), we may want to transmit also the velocity and the pressure. This corresponds to the CC (7) expressed in primitive variables

$$\mathbf{v} = (\tau, v, p)^T. \tag{13}$$

The change of variables $\mathbf{u} = (\tau, v, e)^T = \varphi_\alpha(\mathbf{v})$, is defined assuming that the functions $p = p_\alpha(\tau, \varepsilon)$ may be inverted in $\varepsilon = \varepsilon_\alpha(\tau, p)$, which is the case for instance for an ideal polytropic gas satisfying a γ-law $p = (\gamma - 1)\varepsilon/\tau$, more generally, we assume $\frac{\partial p}{\partial \varepsilon} > 0$.

3.1 Coupling with transmission of primitive variables

The Riemann problem for (11) is usually solved using primitive variable because the 'projection' of the wave curves on the (v, p)-plane are easily expressed. Let $\mathbf{u}_L$ and $\mathbf{u}_R$ be two given states. We denote by $\mathcal{S}_R^1(\mathbf{u}_L)$ the $1-$wave curve consisting of states $\mathbf{u}$ which can be connected to $\mathbf{u}_L$ on the right by either a $1-$shock or a $1-$rarefaction wave corresponding to the equation of state $p = p_R(\tau, \varepsilon)$. Similarly, given a right state $\mathbf{u}_R$, we denote by $\mathcal{S}_L^3(\mathbf{u}_R)$, the (backward) 3-wave curve consisting of left states $\mathbf{u}$ which can be connected to $\mathbf{u}_R$ by a $3-$shock or a $3-$rarefaction wave corresponding to the equation of state $p = p_L(\tau, \varepsilon)$. We denote by $\mathbf{S}_R^1(\mathbf{v}_L)$ and $\mathbf{S}_L^3(\mathbf{v}_R)$ the 'projections' (in a sense to be precised below) onto the (v, p)-plane of the wave curves $\mathcal{S}_R^1(\mathbf{u}_L)$ and $\mathcal{S}_L^3(\mathbf{u}_R)$ respectively. In fact $\mathbf{S}^i(\mathbf{v}_L)$ is the projection of the $i-$wave curve $\varphi^{-1}(\mathcal{S}^i(\mathbf{u}_L))$ expressed in primitive variables $\mathbf{v} = (\tau, v, p)^T$ on the (v, p)-plane:

$$\varphi^{-1}(\mathcal{S}^i(\mathbf{u}_L)) = \left\{\mathbf{v} = (\tau, v, p)^T; \varphi(\mathbf{v}) \in \mathcal{S}^i(\mathbf{u}_L)\right\}$$

and

$$\mathbf{S}^i(\mathbf{v}_L) = \left\{(v, p); (\tau, v, p)^T \in \varphi^{-1}(\mathcal{S}^i(\mathbf{u}_L))\right\}.$$

Proposition 1. *In the case (13), the coupling conditions (7) are equivalent to*

$$v(0-,t) = v(0+,t),\ p(0-,t) = p(0+,t). \tag{14}$$

The proof consists as for the $p-$system in expressing the CC (7) in terms of solutions of Riemann problems and intersection of the projected wave curves. We assume that the curves $\mathbf{S}_R^1(\mathbf{v}_\ell)$ and $\mathbf{S}_L^3(\mathbf{v}_r)$ intersect at one point at most.

3.2 Transmission of conservative variables

In this case, the $(v,p)-$ plane is not well suited, since p is no longer a transmitted variable. For two $\gamma-$laws

$$p_\alpha = (\gamma_\alpha - 1)\varepsilon/\tau \tag{15}$$

we can think of the plane $(v, \pi = \varepsilon/\tau)$, since π is a variable independent of the pressure law. Indeed, following the above arguments while projecting on the $(v,\pi)-$plane, we can prove

Proposition 2. *Assuming (15), the coupling conditions (5) are equivalent to*

$$\left\{ \begin{array}{l} v(0-,t) = v(0+,t), \\ \dfrac{\varepsilon}{\tau}(0-,t) = \dfrac{\varepsilon}{\tau}(0+,t). \end{array} \right. \tag{16}$$

We can easily extend the result to the case of pressure laws which can be written as a function of one dependent variable $\pi = \pi(\tau,\varepsilon)$ i.e. such that $p_\alpha(\tau,\varepsilon) = \overline{p}_\alpha(\pi(\tau,\varepsilon))$. The above argument will show that (v,π) is continuous at the interface $x = 0$. For general pressure laws, the velocity need not be continuous. This is in particular the case for two pressure laws of Grüneisen type

$$p_\alpha(\tau,\varepsilon) = (\gamma_\alpha - 1)\frac{\varepsilon}{\tau} + c_\alpha^2(\frac{1}{\tau} - \frac{1}{\tau_{ref,\alpha}}),\ \alpha = L, R. \tag{17}$$

such that $\frac{c_L^2}{\gamma_L - 1} \neq \frac{c_R^2}{\gamma_R - 1}$ (for details, we refer to [5]).

4 Coupling Lagrangian systems of different dimensions

We consider the $p-$system (9) in the left half-plane and the Euler system in Lagrangian coordinates (11) in the right half-plane (using in this section capital letters to distinguish the conservative variables)

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial}{\partial x}\mathbf{f}(\mathbf{u}) = \mathbf{0}, x < 0,\ \mathbf{u} = (\tau, v)^T,\ \mathbf{f}_L(\mathbf{u}) = (-v, p)^T, p = p_L(\tau)$$

and

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial}{\partial x}\mathbf{F}_R(\mathbf{U})=\mathbf{0}, x>0, \mathbf{U}=(\tau,v,e)^T, \mathbf{F}_R(\mathbf{U})=(-v,p,pv)^T, p=p_R(\tau,\varepsilon).$$

The dimensions of the two systems are now different, but the physical context helps to give a meaning to the coupling since some state variables such as the specific volume τ, velocity v or pressure p are defined for each model. We write the CC using the variables (v,p) that are common to the two systems and which we have seen are good candidates for both. We reconstruct the missing variable for the smaller system in such a way that we may transmit the velocity and the pressure. Indeed, we can lift $\mathbf{v}=(v,p)^T$ by reconstructing τ when we transmit from the left to the right

$$\mathbf{v}=(v,p)^T \rightarrow \mathcal{L}(\mathbf{v})=(\tau,v,p)^T, \tau=\tau_L(p), \tag{18}$$

where $p \rightarrow \tau_L(p)$ is the inverse of $p_L(\tau)$. And we easily project $\mathbf{V}$ when we transmit from the right to the left

$$\mathbf{V}=(\tau,v,p)^T \rightarrow \mathcal{P}(\mathbf{V})=(v,p)^T. \tag{19}$$

Using the previously defined admissible change of variables φ_α, the CC naturally writes

$$\begin{cases} \mathbf{u}(0-,t) \in \mathcal{O}_L(\varphi_L(\mathcal{P}(\mathbf{V}(0+,t)) \\ \mathbf{U}(0+,t) \in \mathcal{O}_R(\varphi_R(\mathcal{L}(\mathbf{v}(0-,t)). \end{cases} \tag{20}$$

We obtain the following result.

Proposition 3. *Assuming (18) with (19), the coupling conditions (20) are equivalent to*

$$\begin{cases} v(0-,t)=v(0+,t), \\ p(0-,t)=p(0+,t). \end{cases} \tag{21}$$

5 Conclusion

The extension of the previous approach to general Lagrangian systems requires some technical developments but is straightforward and presented in [5]. This work is part of an ingoing joint research program on multiphase flows between CEA and University Pierre et Marie Curie. Other topics encountered in the context of the coupling of two-phase flow models are developed in [2] [3] [4].

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