so that

$$\begin{cases} \nabla\lambda_1 \cdot r_1 = -\rho \dfrac{\partial c}{\partial \rho} - c = -\left(c + \rho \dfrac{\partial c}{\partial \rho}\right), \\ \nabla\lambda_2 \cdot r_2 = 0, \\ \nabla\lambda_3 \cdot r_3 = c + \rho \dfrac{\partial c}{\partial \rho}. \end{cases} \qquad (21)$$

Using (19)', we then obtain that the first and the third characteri fields are genuinely nonlinear while the second one is linearly d nerate.

<u>Riemann Invariants</u>

For each characteristic field, we seek $(p-1) = 2$ Riemann invar whose gradients are linearly independent. Recall that a k-Rieman invariant $w : \Omega \to \mathbb{R}$ is defined by

$$\nabla w(u) \cdot r_k(u) = 0 \quad \forall u. \qquad (22)$$

We choose here again to work with the set of variables (ρ, u, s). Let us begin with the first characteristic field. (22) gives

$$\rho \frac{\partial w}{\partial \rho} - c \frac{\partial w}{\partial u} = 0$$

Then, s and $u + l(\rho, s)$ with l defined up to an additive function of s by

$$\frac{\partial l}{\partial \rho}(\rho, s) = \frac{c(\rho, s)}{\rho}$$

are clearly candidates. It remains to check that there gradie are linearly independent. We have

$$\nabla s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \nabla(u + l) = \begin{pmatrix} c/\rho \\ 1 \\ 0 \end{pmatrix}$$

These vectors are clearly independent.

In a similar way, we get that s and $u - l(\rho, s)$ are two Riemann invariants with linearly independent gradients associate with the third characteristic field.